Answers to written re-exam
at the Department of Economics winter 2018-19

Economics of the Environment, Natural Resources and Climate Change

Final re-exam

12 February 2019

(3 hour closed book exam)

Answers only in English

This exam question consists of X pages in total, including this front page.

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Exercise 1. Bioenergy, fossil energy and the greenhouse effect (indicative weight: 3/4)

Wood-based biomass is still the most important source of renewable energy. In the official accounts for emission of greenhouse gases, biomass is considered to be carbon neutral, since it is assumed that the CO_2 emitted when biomass is burned is offset by the sequestration of carbon via the natural growth of the biomass in existing forests. For this reason the use of biomass is typically exempt from taxes on CO_2. This exam exercise asks you to analyze whether such a policy is optimal, using a simplified model of the macro economy and climate change with the following notation:

Y = gross output of final goods before damage from climate change
F = input of fossil energy
B = input of bioenergy
E = total input of energy
c = total cost of producing fossil energy
b = total cost of producing bioenergy
y = net output of final goods available for consumption and investment
X = stock of biomass
S = stock of CO_2 accumulated in the atmosphere
D = output lost due to damages from climate change
g = natural growth of the stock of biomass
W = social welfare
ρ = rate of time preference (constant)
t = time

Final goods are produced by using inputs of energy and a fixed factor of production which is not stated explicitly since it is in constant supply. Hence the gross output of final goods is given by the following production function displaying a positive but declining marginal productivity of energy input:

$$Y_t = f(E_t), \qquad f' > 0, \quad f'' < 0. \tag{1}$$

Fossil energy and bioenergy are assumed to the perfect substitutes in production, so

$$E_t = F_t + B_t. \tag{2}$$

The total cost of producing fossil energy is measured in units of the final good, and the marginal production cost is positive and increasing:

$$c_t = c\left(F_t\right), \qquad c' > 0, \quad c'' > 0. \tag{3}$$

Similarly, we have the following cost function describing the total cost of producing bioenergy (by cutting timber):

$$b_t = b\left(B_t\right), \qquad b' > 0, \quad b'' > 0. \tag{4}$$

Accounting for the costs of energy production and the damage costs of climate change, the net output available for consumption and investment is

$$y_t = Y_t - c_t - b_t - D_t. \tag{5}$$

Reflecting the greenhouse effect, the damage costs of climate change are an increasing function of the stock of carbon accumulated in the atmosphere, with a non-declining marginal damage cost:

$$D_t = D\left(S_t\right), \qquad D' > 0, \quad D'' \geq 0. \tag{6}$$

We may choose our units of measurement such that the burning of one unit of energy (fossil energy or bioenergy) emits one unit of CO_2. We may also measure the natural growth of the stock of wood-based biomass (g) in units of carbon sequestered as a result of the growth of trees. For simplicity, we will abstract from the fact that some of the carbon accumulated in the atmosphere gradually disappears over time, since this happens very slowly. The stock of carbon in the atmosphere will then evolve in the following way, where a dot above a variable indicates its rate of change over time:

$$\dot{S}_t = F_t + B_t - g\left(X_t\right), \qquad g' \geq 0. \tag{7}$$

The specification of the growth function $g\left(X_t\right)$ reflects an assumption that, over the relevant range of values of the biomass stock X_t, the natural growth of the stock of biomass (measured in units of carbon sequestered) increases with the stock, as larger forests are able to absorb more carbon from the atmosphere.

The actual growth of the biomass stock equals the natural growth minus the amount of biomass harvested for energy use:

$$\dot{X}_t = g\left(X_t\right) - B_t. \tag{8}$$

The most abundant type of fossil fuel is coal. The existing global coal reserves are so large that a serious scarcity of coal is a very remote prospect. We will therefore neglect the fact that, ultimately, coal is an exhaustible resource. Instead we will treat the fossil energy in our model as a reproducible input factor, so we do not need to account for the evolution of the remaining reserves of coal.

Finally, we will measure social welfare by the present value of the current and future volume of final goods available for consumption and investment. Applying a constant discount rate ρ, this present value is

$$W_0 = \int_0^\infty y_t e^{-\rho t} dt, \qquad \rho > 0. \tag{9}$$

This completes the description of technology and preferences in the economy.

Question 1.1. A benevolent social planner will choose the time path of energy use so as to maximize the social welfare function (9), subject to the constraints given by eqs. (1) through (8), taking the initial values of the stocks of carbon and biomass (S_0 and X_0) as given. Show that the current-value Hamiltonian (H) for the solution to this optimal control problem can be written as follows, where λ and η are the shadow values of S and X, respectively, and where we have dropped the time subscripts for convenience:

$$H = f\left(F+B\right) - c\left(F\right) - b\left(B\right) - D\left(S\right) + \lambda\left[F + B - g\left(X\right)\right] + \eta\left[g\left(X\right) - B\right]. \tag{10}$$

Indicate the control variables and the state variables in this problem. Is λ a positive or a negative number?

Answer to Question 1.1: The current-value Hamiltonian has the general form

$$H = y + \lambda \dot{S} + \eta \dot{X}. \tag{i}$$

Inserting eqs. (1) through (4) in (5), we have

$$y = f(F + B) - c(F) - b(B) - D(S). \tag{ii}$$

Substituting (ii) along with (7) and (8) into (i) yields eq. (10). The control variables are F and B, and the state variables are S and X. The shadow value λ reflecting the social value of a marginal increase in the atmospheric carbon stock is negative, since a higher carbon stock enhances the greenhouse effect, thereby increasing the damage costs D which detract from the resources y available for consumption and investment.

Question 1.2. Derive the first-order conditions for the social planner's optimal choice of the variables F and B. Provide an economic interpretation of these conditions.

Answer to Question 1.2: From (10) we obtain the following first-order conditions for the socially optimal choice of the control variables F and B:

$$\frac{\partial H}{\partial F} = 0 \quad \Longrightarrow \quad f'(F + B) = c'(F) - \lambda, \tag{iii}$$

$$\frac{\partial H}{\partial B} = 0 \quad \Longrightarrow \quad f'(F + B) = b'(B) - \lambda + \eta. \tag{iv}$$

The left-hand side of (iii) and (iv) is the marginal benefit from energy use, i.e., the increase in output resulting from the use of an extra unit of energy.

The right-hand side of (iii) is the marginal social cost of using fossil fuel, consisting of the marginal production cost plus the social cost of carbon ($-\lambda > 0$), defined as the social cost of emitting an extra unit of CO_2 to the atmosphere (thereby augmenting the atmospheric carbon stock by an extra unit).

The right-hand side of (iv) is the marginal social cost of using bioenergy, consisting of the marginal production cost plus the social cost of carbon and plus the marginal social cost η of reducing tomorrow's biomass stock by harvesting an extra unit today.

Thus the first-order conditions (iii) and (iv) state that, in a social optimum, the marginal social benefit from energy use must equal the marginal social cost, including the cost to the environment.

Question 1.3. Write down the co-state equations describing the socially optimal evolution of the shadow values of the stocks of atmospheric carbon and biomass, respectively (the equations for $\dot{\lambda}$ and $\dot{\eta}$ along the economy's optimal time path). Show that the equation for $\dot{\lambda}$ implies that

$$\lambda_t = -\int_t^{\infty} D'(S_u)\, e^{-\rho(u-t)} du. \tag{11}$$

(Hint: You may use Leibniz' Rule to prove the result in (11)). Explain the economic intuition for this result.

Answer to Question 1.3: From the description of the optimal control problem in Question 1.1 it follows that the relevant co-state equations are

$$\dot{\lambda}_t = \rho\lambda_t - \frac{\partial H_t}{\partial S_t} \quad \Longrightarrow \quad \dot{\lambda}_t = \rho\lambda_t + D'(S_t), \tag{v}$$

$$\dot{\eta}_t = \rho\eta_t - \frac{\partial H_t}{\partial X_t} \quad \Longrightarrow \quad \dot{\eta}_t = [\rho - g'(X_t)]\,\eta_t + \lambda_t g'(X_t). \tag{vi}$$

To prove the result in (11), we apply Leibniz' Rule which says that a function of the form

$$\lambda(t) = \int_{z(t)}^{v(t)} f(t,u)\, du$$

has the derivative

$$\lambda'(t) \equiv \dot{\lambda}_t = f(t, v(t))\, v'(t) - f(t, z(t))\, z'(t) + \int_{z(t)}^{v(t)} \frac{\partial f(t,u)}{\partial t} du \tag{vii}$$

In the present case we have from (11),

$$\lambda(t) = -\int_t^{\infty} D'(S(u))\, e^{-\rho(u-t)} du,$$

so

$$f(t,u) = -D'(S(u))\, e^{-\rho(u-t)}, \qquad z(t) = t, \qquad v(t) = \infty.$$

From this it follows that

$$z'(t) = 1, \qquad v'(t) = 0, \qquad \frac{\partial f(t,u)}{\partial t} = -\rho D'(S(u))\, e^{-\rho(u-t)}, \tag{viii}$$

which may be inserted into formula (vii) to give

$$\begin{aligned}
\lambda'(t) \;&\equiv\; \dot{\lambda}_t = D'(S(t)) - \int_t^{\infty} \rho D'(S(u))\, e^{-\rho(u-t)} du \\
&= D'(S(t)) - \rho \overbrace{\int_t^{\infty} D'(S(u))\, e^{-\rho(u-t)} du}^{=-\lambda(t)} \\
&= \rho\lambda(t) + D'(S(t)).
\end{aligned} \tag{ix}$$

We see that the last line in (ix) is identical to the right-hand side of (v). This proves that the first-order condition (v) does indeed imply the expression for the shadow price λ_t stated in (11).

According to (11) the (numerical) shadow price of the carbon stock S_t (the social cost of carbon, $-\lambda_t$) equals the present value of the current and future output losses generated by a unit increase in the current carbon stock.

(Note: Question 1.3 is difficult, as many students will not be able to remember Leibniz' Rule, so only the best students will be able to answer this question in a fully satisfactory manner).

We will now consider a market economy with the technology and preferences described by eqs. (1) through (9). The market price of energy will be denoted by p and will be measured relative to the price of final goods which is our numeraire good with a price of 1. The production side of the market economy consists of three sectors: A final goods sector, a sector producing fossil-based energy, and a sector producing wood-based bioenergy. There are many identical firms in each sector, so perfect competition prevails throughout the economy. All firms take the climate and thereby the damage cost $D(S)$ as given.

The production technology of the representative final goods producer is given by (1) and (2). In period t, the net cash flow generated by the final goods producer is $Y_t - p_t E_t - D(S_t) = f(F_t + B_t) - p_t(F_t + B_t) - D(S_t)$. The final goods firm chooses its energy inputs so as to maximize the present value V_0^Y of the current and future net cash flows to its owners. When maximizing this present value given by

$$V_0^Y = \int_0^\infty \left[f(F_t + B_t) - p_t(F_t + B_t) - D(S_t)\right] e^{-\rho t} dt, \tag{x}$$

the firm takes the energy price p_t as well as $D(S_t)$ as given.

The representative producer of fossil-based energy sells it output F_t to the final goods sector. The government levies a carbon tax at the rate of τ_t^F per unit of fossil fuel sold, so the net cash flow earned by the fossil fuel producer is $\left(p_t - \tau_t^F\right) F_t - c(F_t)$ per period, where $c(F_t)$ is given by (3). The producer of fossil energy chooses its volume of output F_t with the purpose of maximizing the present value V_0^F of the net cash flows paid out to its owners, taking the energy price and the carbon tax as given. This present value is

$$V_0^F = \int_0^\infty \left[\left(p_t - \tau_t^F \right) F_t - c\left(F_t\right) \right] e^{-\rho t} dt. \tag{xi}$$

The representative producer of bioenergy likewise sells its output to the final goods sector. The bioenergy firm owns the forest from which it harvests the timber for energy use. As part of its climate policy, the government may choose to pay a "conservation subsidy" at the rate s_t^B per unit of carbon stored in the forest biomass. The government may also levy a carbon tax at the rate τ_t^B per unit of biomass (measured in tons of carbon) harvested from the forest for energy use. In period t the bioenergy firm therefore earns a net cash flow to its owners equal to $\left(p_t - \tau_t^B \right) B_t - b\left(B_t\right) + s_t^B X_t$, where the cost function $b\left(B_t\right)$ is given by (4). The present value V_0^B of this cash flow is

$$V_0^B = \int_0^\infty \left[\left(p_t - \tau_t^B \right) B_t - b\left(B_t\right) + s_t^B X_t \right] e^{-\rho t} dt. \tag{xii}$$

Question 1.4. Derive the first-order condition for the final goods firm's privately optimal input of energy $E_t = F_t + B_t$. Give an economic interpretation of this first-order condition. (Hints: Note that since F_t and B_t are perfect substitutes, the final goods firm is indifferent between using fossil energy and bioenergy, so the first-order condtion for the use of the two forms of energy is the same. Notice also from (x) that the present value of the firm's cash flow will be maximized if it maximizes its net cash flow in each single time period t, so you do not need to use optimal control theory to find the optimum, since the firm takes $D\left(S_t\right)$ as given).

Answer to Question 1.4: For a given value of $D\left(S_t\right)$, it follows from (x) that the final goods producer will maximize its market value V_0^Y by maximizing the term $f\left(F_t + B_t\right) - p_t\left(F_t + B_t\right)$ with respect to $F_t + B_t$ for all $t \geq 0$. The first-order condition for the solution to this maximization problem is

$$f'\left(F_t + B_t\right) = p_t. \tag{xiii}$$

Eq. (xiii) states that, at the firm's optimum, the (value of the) marginal productivity of energy must equal its price. We note that since fossil energy and bioenergy have the same marginal productivity, the final goods firm will be indifferent between them, so the firm's energy mix will be determined from the supply side of the energy market.

Question 1.5. Derive the first-order condition for the privately optimal volume of F_t supplied by the producer of fossil energy. Give an economic interpretation of this first-order condition. (Hint: Note from (xi) that the present value of the firm's cash flow will be maximized if it maximizes its net cash flow in each single time period t, so you do not need to use optimal control theory to find the optimum).

Answer to Question 1.5: From (xi) we see that the market value V_0^F of the fossil energy producer is maximized when each period's cash flow $\left(p_t - \tau_t^F\right) F_t - c\left(F_t\right)$ is maximized with respect to F_t. The first-order condition for the solution to this problem is

$$c'\left(F_t\right) + \tau_t^F = p_t. \tag{xiv}$$

Hence the private optimum is reached when the private marginal cost of producing fossil energy (including the carbon tax) equals the market price at which the energy can be sold.

Question 1.6. Since the bioenergy firm owns the forest, it has an incentive to account for the impact of its current harvest of timber on the evolution of the forest biomass determined by eq. (8). The firm will therefore wish to maximize its present value (xii) subject to the stock-flow constraint (8), given the pre-determined initial biomass stock X_0. Use optimal control theory to derive the first-order condition for the bioenergy firm's privately optimal choice of the harvest rate B_t (the firm's control variable) as well as the condition for the privately optimal rate of change over time of the shadow value of the firm's biomass stock, denoted by η_t^B. Provide an economic interpretation of the condition for the optimal choice of B_t.

Answer to Question 1.6: From (8) and (xii) it follows that the current-value Hamiltonian H_t^B corresponding to the bioenergy firm's optimal control problem is

$$H_t^B = \left(p_t - \tau_t^B\right) B_t - b\left(B_t\right) + s_t^B X_t + \eta_t^B \left[g\left(X_t\right) - B_t\right]. \tag{xv}$$

From (xv) we obtain the first-order conditions

$$\frac{\partial H_t^B}{\partial B_t} = 0 \quad \Longrightarrow \quad b'\left(B_t\right) + \tau_t^B + \eta_t^B = p_t, \tag{xvi}$$

$$\dot{\eta}_t^B = \rho \eta_t^B - \frac{\partial H_t^B}{\partial X_t} \quad \Longrightarrow \quad \dot{\eta}_t^B = \left[\rho - g'\left(X_t\right)\right] \eta_t^B - s_t^B. \tag{xvii}$$

Condition (xvi) states that the bioenergy firm's privately optimal harvest rate B_t is attained when the private marginal cost of harvesting an extra unit of biomass (the left-hand side) equals the market price at which it can be sold. The private marginal cost consists of the marginal production cost, $b'(B_t)$, plus the carbon tax rate on bioenergy, τ_t^B, plus the opportunity cost η_t^B of reducing the future stock of biomass by harvesting an extra unit today rather than tomorrow.

Question 1.7. Use your previous results to derive the values of the tax and subsidy rates τ_t^F, τ_t^B and s_t^B which will ensure that the market economy will follow the first-best socially optimal time path. (Hint: Find the values of τ_t^F, τ_t^B and s_t^B that will make the first-order conditions for the market economy coincide with the social planner's first-order conditions). Explain the economic intuition for your results. Discuss briefly whether it is realistic to expect that the government can implement the optimal set of taxes and subsidies in practice.

Answer to Question 1.7: Using (xiii) to elimate p_t from (xiv) and (xvi), we get

$$f'(F_t + B_t) = c'(F_t) + \tau_t^F, \tag{xviii}$$

$$f'(F_t + B_t) = b'(B_t) + \tau_t^B + \eta_t^B. \tag{xix}$$

Now suppose that the tax rates and the subsidy rate in the market economy are set at the following levels:

$$\tau_t^F = \tau_t^B = -\lambda_t, \tag{xx}$$

$$s_t^B = -\lambda_t g'(X_t). \tag{xxi}$$

Inserting (xx) in (xviii) and (xix), we obtain

$$f'(F_t + B_t) = c'(F_t) - \lambda_t, \tag{xxii}$$

$$f'(F_t + B_t) = b'(B_t) - \lambda_t + \eta_t^B. \tag{xxiii}$$

Next we insert the expression (xxi) for the subsidy to biomass conservation into the bioenergy firm's optimum condition (xvii) to find that

$$\dot{\eta}_t^B = [\rho - g'(X_t)]\,\eta_t^B + \lambda_t g'(X_t). \tag{xxiv}$$

Comparing the social planner's optimum conditions (iii), (iv) and (vi) to the optimality conditions (xxii), (xxiii) and (xxiv) for the market economy, we see that the two sets of conditions have exactly the same structure and will be identical if $\eta_t^B = \eta_t$. Intuitively, this equality must hold since the planned economy and the market economy start out with the same initial stocks S_0 and X_0 (and assuming a unique optimum solution for both economies). Hence the climate policy specified in (xx) and (xxi) will ensure that the market economy follows the socially optimal path of energy use. (*Note: This is not a rigorous mathematical proof, but since this is a challenging question, it is quite satisfactory if the student answers along the lines suggested above*).

The intuition behind the tax rule (xx) is that a carbon tax equal to the social cost of carbon $(-\lambda_t)$ will induce energy producers to internalize the external damage cost of climate change in their decisions. The conservation subsidy in (xxi) equals the marginal social benefit from a lower damage cost of climate change as the conservation of an extra unit of biomass reduces the stock of carbon in the atmosphere by the amount $g'(X_t)$ through carbon sequestration. Via this subsidy the marginal external benefit from biomass conservation is internalized in the decisions of bioenergy producers. Note that since there are two different externalities associated with the use of bioenergy (a negative externality from the emission of CO_2 when the biomass is burned and a positive externality when the stock of biomass grows), the government needs to use two different policy instruments (τ^B and s^B) to internalize the externalities.

In practice it is unlikely that the government possesses the information needed to implement the optimal climate policy specified in (xx) and (xxi) in an exact way. For one thing, there is great uncertainty about the damage function $D(S)$ due to uncertainty about climate sensitivity and climate tipping points. Hence the social cost of carbon given by (11) is also highly uncertain. Furthermore, to calibrate the conservation subsidy specified in (xxi), the government would have to be able to measure the biomass stock X_t and its marginal impact on the natural growth of the stock. This would also raise considerable measurement problems.

At the same time, however, the analysis in this question strongly suggests that the current policy of exempting wood-based bioenergy from carbon taxes is inoptimal.

Exercise 2. Principles of optimal environmental taxation (indicative weight: 1/4).

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful).

Question 2.1: Explain what determines the optimal tax on a polluting good in a first-best setting where the government can use lump-sum taxes and subsidies.

Answer to Question 2.1: When the government can meet its needs for revenue and income redistribution through lump-sum taxes and subsidies, it does not have to use any distorting taxes and transfers. Assuming there are no other market imperfections, the government's environmental tax policy may then concentrate on internalizing the externalities from the emission of pollutants caused by the consumption of a polluting good. This can be done by levying a Pigouvian tax on emissions corresponding to the marginal external cost of emissions. The marginal external cost is given by the sum of all individual consumers' marginal willingness to pay for a reduction in the flow of emissions.

Question 2.2: Explain what determines the optimal tax on a polluting good when the government cannot use lump sum taxes and has to raise a given amount of revenue from distorting indirect taxes on consumer goods.

Answer to Question 2.2: In this setting the government must account for the fact that an increase in taxes on polluting goods will exacerbate the distortions to consumption and labour supply caused by the existing indirect consumption taxes. Assuming for the moment that the government only cares about economic efficiency and that consumer utility functions are quasi-linear in consumption and leisure (implying zero cross-price elasticities in the consumption of different consumer goods), one can show that the second-best optimal tax on a polluting good then becomes a weighted average of a "Pigou term" and a "Ramsey term". The Pigou term is the marginal external cost of the pollution generated by consumption of the good, i.e., the first-best optimal Pigouvian tax. The size of the Ramsey term is inversely related to the price elasticity of demand for the taxed good. The reason is that a higher price elasticity implies a larger distortion to labour supply as consumers substitute to a larger extent away from the taxed good towards more consumption of leisure. Ceteris paribus, the optimal tax on a polluting good will thus be lower than the marginal external cost to the environment if the price elasticity of demand for the good is high. The weight of the Ramsey term in the optimal tax formula will be larger the greater the government's need for revenue, since the collection of a larger revenue requires a higher level of indirect taxes in which case the government must pay more attention to the need for minimizing tax distortions to labour supply.

When the government cares about equity (income distribution) as well as efficiency, the optimal tax rate on a polluting good will depend on its weight in the budgets of different income groups as well as on its price elasticity of demand and on the marginal external cost of pollution generated by consumption of the good. If the good is consumed mainly by high-income earners, the optimal tax rate may exceed the first-best Pigouvian level, but if it is consumed mainly by households with low incomes, the optimal tax rate is likely to be lower than the Pigouvian tax due to concerns about the distributional effects. Moreover, when distributional concerns are included, the low-income earners' marginal willingness to pay for an improved environment is given greater weight than the high-income earners' marginal willingness to pay when calculating the marginal external cost of pollution in the optimal tax formula.

However, one can argue that the government should not let the tax rates on polluting goods be affected by concerns about revenue raising or income distribution since the progressive personal income tax is a more effective and well-targeted instrument for raising revenue and redistributing income.